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Multiple time scales and subsystem embedding in the learning of juggling

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Abstract

To gain insight into the multiform dynamics and integration of remote yet pertinent subsystems into the performance of complex perceptual–motor skills, we recently conducted a series of longitudinal and cross-sectional experiments on the acquisition of 3-ball cascade juggling in which we measured, next to the ball trajectories, postural sway, eye and head movements and respiration. The aim of the present paper is to review the main results and theoretical implications of these experimental studies for understanding skill acquisition. As regards the evolution of the quality of the juggling itself, we found that only certain aspects of throwing and catching were adjusted, while the goal behavior of sustained juggling (operationalized as the number of consecutive throws) and the degree of frequency and phase locking between the ball trajectories, indexing pattern stability, increased monotonically. The latter three aspects evolved at different rates, reflecting the existence of a temporal hierarchy in learning. Postural sway exhibited initial manifestations of task-specific, possibly mechanically induced, modes of 3:1 and 3:2 frequency locking with the ball trajectories and only few transitions between those modes. Functional stability appeared to be enhanced during practice by minimizing the sway amplitudes rather than by adjusting the sway dynamics itself. Eye and point-of-gaze movements also showed instances of 3:1 and 3:2 frequency locking with the ball trajectories; especially establishing a 3:1 locking (horizontal eye movements) appeared to be important. Expert behavior suggested that extended practice promotes reliance on multiple sources of information, allowing the proficient juggler to switch adaptively between functional organizations involving distinct perceptual systems. No consistent coordination between

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breathing and juggling was found. It was concluded that multiform dynamics, involving hierarchically ordered time scales, underlie the acquisition of complex skills and that the subsystems subserving realization of the task goal become assembled and embedded in a task- and subsystem-specific manner.

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1. Introduction

Dexterity has been defined as ''the ability to find a motor solution for any external problem, that is, to adequately solve any emerging motor problem correctly, quickly, rationally and resourcefully'' (Bernstein, 1996, p. 228). In other words, dexterity refers to the ability to exert adequate control in a large variety of (unexpectedly) evolving circumstances, implying stable yet flexible motor execution, and further hints at an efficient and effective utilization of the resources of the action system, entailing a functional coordination among multiple pertinent subsystems. In the study of skill acquisition, the issue of control has dominated the research agenda and has typically been addressed in terms of motor redundancy and the corresponding degrees of freedom problem (Bernstein, 1967). That is, researchers have primarily studied the principles underlying the formation of synergies, or coordinative structures, and, as a rule, have focused on effectors that have a direct bearing on task achievement, such as wrist, elbow and shoulder movements in writing (Newell & van Emmerik, 1989) and serving a volleyball (Temprado, Della-Grasta, Farrell, & Laurent, 1997). Notwithstanding the merits of this approach, it has drawn attention away from the issue of resourcefulness, thus ignoring that, in general, goal achievement also depends on a variety of more remote processes subserving performance. For instance, manipulating an object in space requires picking up relevant information, keeping the center of mass above the base of support and liberating and consuming metabolic energy. Thus, when performing a complex skill, a variety of specialized subsystems have to be assembled and embedded into a functional organization. Although no one would probably deny the importance of such processes, systematic examinations of how subsystems contribute to and constrain the goal behavior, as well as changes therein as learning proceeds, are few and far between. This is unfortunate in view of the increasing awareness that the acquisition of perceptual–motor skills is a complex dynamical process that involves changes in multiple systems occurring at multiple time scales (cf. Newell, Liu, & Mayer-Kress, 2001). Clearly, examining skill acquisition form this broader perspective requires sophisticated analyses of the goal behavior itself as well as that of the contributing subsystems. These theoretical considerations motivated us to perform several longitudinal experiments on the learning of 3-ball cascade juggling, as well as a cross-sectional study of expert and intermediately skilled jugglers. In the present article, we review the main results and insights that were obtained in these studies, combined with those of earlier investigations on juggling. In doing so, we attempt to develop an encompassing view on the learning of this particular perceptual–motor skill. Before discussing the results and their theoretical implications for perceptual–motor learning, we will first summarize the key insights into perceptual–motor learning from a dynamical systems perspective and outline a theoretical framework for understanding the assembly and embedding of multiple subsystems into a functional organization.

1.1. Controllability and flexibility of coordinative structures

In the investigation of the formation and controllability of coordinative structures, two lines of research may be distinguished, one focusing on changes in pattern stability and the other on changes in dimensionality. In the first approach, patterns of (usually bimanually) coordinated rhythmic movements are characterized in terms of their relative phase dynamics and learning as a re-parameterization of this dynamics (Schöner, 1989; Schöner, Zanone, & Kelso, 1992; Swinnen, Walter, Lee, & Serrien, 1993; Zanone & Kelso, 1992, 1997). Based on this modeling approach, Schöner et al. (1992) anticipated that early in the learning of an initially unstable bimanual coordination pattern (e.g., 90° relative phase between oscillating hands) performers would produce an intrinsically stable but unintended pattern (e.g., 0°). It was predicted that the to-be-learned pattern would be acquired via a phase transition, and be accompanied by a drastic increase in variability, a phenomenon called critical fluctuations. These predictions were confirmed empirically, with the occurrence of transitions depending on the learner's initial coordination tendencies as well as the to-be-learned coordination (Zanone & Kelso, 1997), thus demonstrating that the formation and stabilization of coordinative structures carry the signs of self-organization.

The second, more descriptive, approach focuses on the dimensionality of (usually cyclical) movement patterns. The dimensions in question, which may take the form of components or number of active degrees of freedom, are assumed to reflect the underlying control structure and changes therein due to learning. Methodological differences notwithstanding (principle component analysis versus state-space reconstruction), both Haas (1995; see also Haken, 1996) and Mitra, Amazeen, and Turvey (1998) found a reduction in dimensionality in the course of learning. These findings led to the suggestion that learning might be characterized by reduced dimensionality, at least during its early and intermediate stages. The implicit assumption that control and dimensionality are inversely related may be premature, however, as it was recently shown that reduced dimensionality also seems to be a characteristic of movement disorders and impairments (cf. Longstaff & Heath, 2003; Newell & Vaillancourt, 2001), and that the dimensionality of movement depends in part on the prevailing task constraints (Newell, Broderick, Deutsch, & Slifkin, 2003).

Recently, the control structure underlying coordinative structures has also been examined through the concept of the uncontrolled manifold (Scholz & Schöner, 1999; Scholz, Schöner, & Latash, 2000; Schöner, 1995), which emphasizes the flexible exploitation of redundant degrees of freedom rather than their elimination. The gist of this approach is that the variability in the high-dimensional space of joint configurations can be divided into two subspaces. Whereas the variability associated with one subspace does not correlate to task achievement, the variability in the other subspace is essential for task success. Increase of the variability in the former subspace relative to that of the latter, as observed when comparing successful and unsuccessful performances, implies that the "non-essential" variables may be selectively released from control. It remains to be seen, however, to what extent the uncontrolled manifold represents a generic, encompassing concept. Recently, Müller and Sternad (2004) proposed that the mapping from task execution onto task outcome is in fact mediated by three components of variability (stochastic noise, task tolerance and task-specific co-variation), and showed that the relative contribution of these components changes with practice. Collectively, these studies indicate that discovering which variables are essential for success and which are not is likely to be an important aspect of skill acquisition.

1.2. Flexibility between coordinative structures

Nearly all of the aforementioned approaches focused exclusively on end-effectors having a direct bearing on task achievement. In general, however, successful realization of a given task goal requires the simultaneous and sequential performance of a variety of other processes subserving task execution, pertaining to, among others, information pick-up, postural control and energy liberation and consumption. Whereas it is clear that specific subsystems are devoted to, or at least implicated in, such subtasks, systematic investigations of how these subsystems contribute to and constrain the goal behavior as learning progresses are largely lacking. This is unfortunate because the assembly and embedding of subsystems into an effective task-specific organization seems to be an essential ingredient of skill learning, as is testified to by the fact that intriguing and intuitively appealing theoretical ideas concerning this topic have been proposed by several authors (see below). Studying these phenomena and ideas is likely to enhance and broaden the understanding of learning and motor control in general.

In his remarkable treatise on dexterity (which was written in Russian in the early 1940s), Bernstein (1996) pioneered the topic of the assembly and embedding of subsystems during skill learning. He started out from the observation that the behavioral repertoire of animals expanded as new classes of perceptual–motor behavior evolved. With the evolution of these new behaviors new perceptual modalities, effector systems and supporting neural systems evolved interdependently, resulting in a hierarchically organized human nervous system. To account for the functional aspect of this hierarchy, Bernstein distinguished four levels of control in the "construction of movement": the level of tonus (i.e., muscular tone), which is involved in all movements and postures, as well as respiration; the level of synergies (i.e., muscular–articular linkages), which allows for the production of coarse-grained coordination patterns (developed in conjunction with proprioceptive systems); the level of space, which is responsible for movements attuned to the environment (developed in conjunction with exteroceptive perceptual systems); and the level of actions, which is responsible for the planning, sequencing and steering of goal-directed actions in an adaptive and creative fashion. Bernstein's hierarchical notion of the "construction" of movement rests on his proposition that the subserving lower levels are recruited and controlled by a leading level, which may be the level of space or the level of action, depending on the task performed and the degree to which it has been practiced. Bernstein further suggested that, during practice, control is delegated to lower levels as sensory "background" correction mechanisms evolve that reduce the necessity for feedback control via the higher control levels, thus unburdening the latter – a process usually called "automatization". According to Bernstein, skill acquisition thus involves the task-specific delegation and distribution of control over specialized functional subsystems, or, in the parlance of dynamics, the formation and annihilation of coupling functions.

In a similar spirit, albeit with a different emphasis, Newell et al. (2001) recently proposed that in an individual in action, three cross-coupled levels can be distinguished at which processes operate at distinct time scales. Specifically, they distinguished a level of physiological micro-phenomena (e.g., cortical activity), a level of coordination between subsystems (e.g., torso and limb movement patterns), and a level of macro-phenomena pertaining to the evolving performance itself. According to Newell et al. (2001), the characteristic times pertaining to these levels are in the order of milliseconds to seconds, minutes to hours and months to years, respectively. In fact, given that Bernstein's (1996) distinction of levels in the "construction of movement" is essentially based on phylogenetic considerations, it may well be interpreted in this spirit. Newell et al. (2001) further proposed that the explicit form of change at the level of performance (the outcome variable) in the course of learning is the product of multiple, dynamically distinct processes.

Whereas Newell et al. (2001) emphasized that (changes in) performance should be understood through the nested dynamics of the action system, Bingham (1988) distinguished between the inherent dynamics, the collection of (usually non-linear) properties of all the components of the action system, and the incidental dynamics brought about by the task at hand. He proposed that the "global" task-organization is defined over both types of dynamics and that the action system wields its own overall dynamics so as to achieve an, at least locally, optimal structure.

The upshot of these considerations is that the learning of complex perceptual–motor tasks is a multi-dimensional process in which functional linkages between various contributing subsystems may form or annihilate in a task-specific manner. Intriguing, and essentially unresolved, questions in this regard concern the principles by which coordinative patterns are formed and annihilated, how these coordinative patterns affect performance, whether the resources of the human action system evolve at similar or distinct time scales, and, in case the anticipated changes are ordered temporally, the principles through which such order comes about.

2. A brief overview of the experiments

In this section, we provide a brief overview of the experiments we conducted on the topic of interest. Given the purpose of the present paper, we here only provide a brief explanation of some of the analysis techniques used, in so far as such an explanation is essential to understand the results obtained. For detailed explanations of the methods and procedures used, we refer to the original experimental papers.

In order to quantify the frequency locking ratio between subsystems, we examined the power spectral densities $P(\omega)$ and identified the dominant frequency ω_0 at the peak containing the most spectral power. Eventual cross-relations between different signals were quantified in terms of the ratio between their dominant frequencies, and a p:q frequency locking between signals was considered to be present if

$$\frac{(p \cdot \omega_{0,x} - \Delta\omega)}{(q \cdot \omega_{0,y} + \Delta\omega)} \leqslant \frac{\omega_{0,x}}{\omega_{0,y}} \leqslant \frac{(p \cdot \omega_{0,x} + \Delta\omega)}{(q \cdot \omega_{0,y} - \Delta\omega)}$$

where $\omega_{0,x}$ and $\omega_{0,y}$ denote the main frequencies of (arbitrary) time-series $x(t)$ and $y(t)$, respectively, and $\Delta\omega$ is the frequency resolution of the corresponding spectral estimates. The strength of frequency locking between the signals was quantified by means of

$$\psi_{x,y}(\rho) = 2n \frac{\int P_x(\omega) \cdot P_y(\rho \cdot \omega)\,\mathrm{d}\omega}{\int \left[P_x^2(\omega) + P_y^2(\rho \cdot \omega)\right] \mathrm{d}\omega}$$

where $P_x(\omega)$ and $P_y(\omega)$ represent the spectral densities of the (arbitrary) time-series $x(t)$ and $y(t)$. n is a normalization factor, which corrects for the systematic deviation of the spectral estimate P_y due to the stretching of the frequency axis as a function of ρ, defined as $8n^2 = (\rho^2 + 1)/(\rho + 1)$. By fixing the ratio ρ at a specific value, one can always determine the strength of a specific frequency locking at this ratio, regardless of whether or not this locking dominates the coordination between the two time-series. The more similar two spectra are after an appropriate rescaling ρ of the frequency axis, the larger their overlap and thus the higher the value of $\psi_{x,y}$ (ρ: see Daffertshofer, Peper, Frank, & Beek, 2000). Recall that the spectral density estimate of a time-series scales with the time series' variance. Thus, when calculating the frequency locking strength between two time-series at a specific ratio after normalization to unit variance, the locking strength exclusively reflects their degree of coordination (in the spectral domain). In the following, we will refer to this variance- or amplitude-independent locking strength as "pure coordination", using the symbol $\psi_{x,y}^{\mathrm{pc}}$. However, scaling $P_x(\omega)$ at the dominant frequency to one and omitting normalization of $y(t)$ (or, equivalently $P_y(\omega)$), the frequency locking strength at ρ also reflects the amplitude of the oscillations of $y(t)$ at this specific frequency locking ratio. In this case we speak of "amplitude effects", using the symbol $\psi_{x,y}^{\mathrm{amp}}$. To anticipate, if the locking strength in terms of pure coordination remains effectively constant during practice, changes in the amplitude effects solely reflect changes in the amplitude of the frequency locked oscillations. [1]

Except for the study of Huys and Beek (2002), in which a point estimate of relative phase was used, we always used the Hilbert transform to compute the phase of the time-series of interest. The relative phase $\Delta\Theta_{\mathrm{H}}(t)$ between two time-series $x(t)$ and $y(t)$ is then defined as $\Delta\Theta_{\mathrm{H}}(t) = \Theta_{\mathrm{H},y}(t) - \Theta_{\mathrm{H},x}(t)$. In cases of $p{:}q$ frequency locking, the latter definition was generalized in terms of $\Delta\Theta_{\mathrm{H}}(t) = p\Theta_{\mathrm{H},y}(t) - q\Theta_{\mathrm{H},x}(t)$. The mean and the variance of the corresponding relative phases were computed using circular statistics (Mardia, 1971). This variance reflects the degree of phase locking, and is denoted $\sigma\Theta$.

We further analyzed the cross-covariance structure of N-dimensional sets of time-series in terms of principal component analysis (PCA; cf., Post, Daffertshofer, & Beek, 2000). That is, we computed the eigenvalues and eigenvectors of the covariance matrix based on the N time-series, which were first normalized. The resulting eigenvalue spectrum was analyzed and, in addition, the time-series were projected onto the individual eigenvectors (referred to as "projections"). These projections were subjected to spectral analysis as described above.

Learning curves were fitted using the function $A - B\mathrm{e}^{-\lambda t}$ and its parameters were determined via a simplex minimization of the corresponding least squares (Nelder & Mead, 1965). The rate of change, which represents the evolution rate, is given by λ yielding characteristic times by means of $\tau = 1/\lambda$.

In Huys, Daffertshofer, and Beek (2003), we examined the development of task-specific couplings between the ball circulations, on the one hand, and respiration and body sway, on the other hand, as six novices learned to juggle the 3-ball cascade for 20 h. Starting at the second day, the juggling movements of the participants were video-recorded every third day up to the 20th day, while simultaneous recordings were made of the respiration and center-of-pressure (CoP) trajectories. Besides determining the spatial and temporal positions of the throws and catches, we quantified the quality of juggling by means of the frequency locking strength and relative phase variance between the ball movements, and its coordination with CoP and respiration in terms of the frequency locking ratio. We further applied a "windowed" PCA to pinpoint the structural changes in the CoP trajectories as a function of practice.

In a follow-up experiment (Huys, Daffertshofer, & Beek, 2004), 13 novices practiced the 3-ball cascade for 9 weeks. In this experiment, ball movements, CoP trajectories, and eye and head movements were recorded simultaneously. Again, frequency locking strength and relative phase variance between ball movements served to quantify juggling performance. To examine the coordination between juggling performance and the activity in the subsystems, we computed the frequency locking

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[1] Note that we always calculated the locking strength relative to the ball movements in the vertical direction; the index x in $\psi_{x,y}$ is therefore further omitted. Thus, for example, $\psi^{\mathrm{amp}}_{\text{AP-sway}}$ denotes the frequency locking strength in terms of amplitude effects between the ball trajectories and AP-sway; $\psi^{\mathrm{pc}}_{\mathrm{balls}}$ denotes the (mean) frequency locking strength in terms of pure (i.e., amplitude-independent) coordination between the (three) paired ball trajectories.

strength with and without rescaling the time-series to unit variance, enabling us to tease apart the amplitude effects from pure coordination effects. In addition, we explicitly studied and compared the rates of change over which the subsystems' coordination patterns evolved by extracting characteristic learning times τ and correlating learning curves.

In Huys and Beek (2002), we focused on the coordination between ball trajectories and point-of-gaze (PG) as a function of juggling tempo, pattern and expertise. For this purpose, intermediately skilled and expert jugglers juggled the standard and the reverse 3-ball cascade at three tempos (slow, preferred and fast) while the ball movements and the PG trajectories were recorded simultaneously. Juggling performance and the coordination between ball movements and PG trajectories were quantified in terms of the ratio and strength of frequency locking. In addition, we quantified the spatial extension of the juggled patterns by calculating the horizontal distance between consecutive zeniths and the height of the throws (the vertical distance traveled by a ball from the location of the throw to its zenith) as well as the amplitudes of the PG trajectories in the both directions using a peak-finding algorithm.

3. Subsystem dynamics and multiple processes in learning to juggle

3.1. The 3-ball cascade juggle

An important feature of juggling (see Fig. 1), which makes it a particularly suitable task for the study of (the learning of) perceptual–motor performance, is that (some of) its task constraints can be formalized. Shannon (cf. Beek & Lewbel, 1995; Horgan, 1990; Raibert, 1986; Sloane & Wyner, 1993) identified the temporal constraints of juggling, known as Shannon's juggling theorem, which relates the number of hands (H) and balls (N) to the time a ball is in flight (t_f), the time a ball is held in hand (t_h) and the time that a hand moves empty (t_e). It dictates that, on average, the ratio between the cycle time of the balls, ($t_f + t_h$) and the cycle time of the hands, ($t_h + t_e$) equals N/H. In other words, it stipulates frequency locking be-

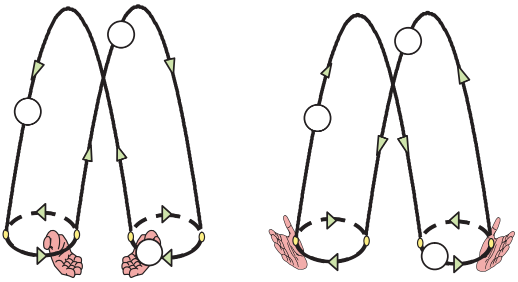

Fig. 1. Schematic representation of the 3-ball standard (left) and reverse cascade (right). The positions of the hands indicate the positions of the throws.

tween the hand and ball movements, and thus between the hand movements as well as between the ball movements. Shannon's juggling theorem may be seen as a formalization of the "global" temporal constraint or macro-structure of juggling (cf. Beek, 1989). A further temporal constraint, which may be seen as the micro-structure of juggling (Beek, 1989), pertains to the patterning of the hand circulation in terms of the dwell time, that is, the time a hand is filled with a ball during the entire hand-cycle time. In examining how jugglers accommodate these temporal constraints, Beek (1989, see also Beek & Turvey, 1992), focusing on the discrete time intervals of juggling, found that specific coordination solutions underlie the patterning of the hand loop. Further investigations of the spatial aspects of juggling revealed that, first, the variability of the angle of ball release was smaller than that of the distance between the throw and its consecutive catch, indicating that the variability of the ball movements accumulates from the throws to the catches, and second, that the variability of variables pertaining to the hand movements was larger than those pertaining to the ball movements (Van Santvoord & Beek, 1996). Instead of focusing on discrete events (which confines the analysis to a limited amount of information selected beforehand), Post et al. (2000) concentrated on the entire structure of the ball pattern as a function of tempo using PCA. They found that, due to symmetries in juggling, in the frontal plane the 6D pattern (3 balls × 2 directions) could be described effectively by two to maximally four dimensions (for the preferred and fast tempo condition, respectively). In fact, these symmetries could be interpreted in terms of frequency and phase locking between the balls, that is, the (degree of) frequency and phase locking determine the dimensionality of the juggling pattern (see the Appendix of Post et al., 2000, for the analytical deduction). Importantly, the degree to which a juggler satisfies these "global" and "local" constraints can be quantified by examining the strength of frequency locking between ball movements and the variance of the relative phase between balls movements, respectively.

3.2. Performance outcome and stability

Consistent with the results of Van Santvoord and Beek (1996), showing that the variability of variables pertaining to the hand movements was larger than those pertaining to the ball movements, we observed that only the spatial variability of the latter decreased in the course of learning (Huys et al., 2003). Similarly, the variability of the moment of catching decreased during learning, whereas the variability of the moment of throwing remained fairly constant, which suggests that the direction and velocity of the consecutive throws became more uniform. Thus, as practice proceeded, only certain discrete events in juggling recurred with increasing regularity. In fact, adjusting these discrete events enhanced the stability of the juggling pattern, as operationalized by the degree of frequency and phase locking between the ball movements ($\psi^{\mathrm{pc}}_{\mathrm{balls}}$ and $\sigma\Theta$, respectively). The evolutions of the variables discussed so far often appeared to have an exponential form, but no attempt was made to rank the characteristic times τ (i.e., the evolution rates) of the observed changes. (Note that in this study the number of data points per individual learning curve ranged from 5 to 7, rendering a quantification of characteristic times rather hazardous.)

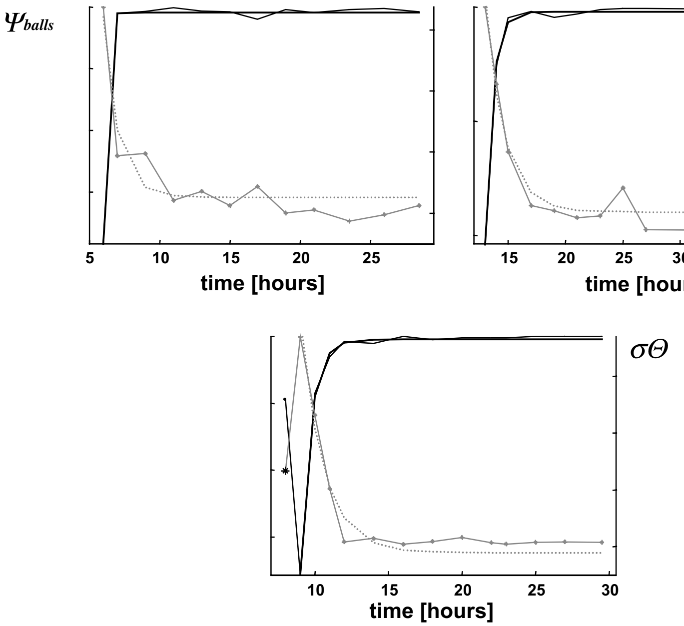

Fig. 2. The experimental fits and data of the evolutions of the frequency locking strength ($\psi^{\mathrm{pc}}_{\mathrm{balls}}$; always on left axis) and variance of the relative phase ($\sigma\Theta$; always on right axis) between ball trajectories. Notice that $\psi^{\mathrm{pc}}_{\mathrm{balls},y}$ changes markedly faster than $\sigma\Theta$ in the left panel and the nearly identical evolutions in the middle panel. An example of an initial deterioration (between the first and third data point) in performance is depicted in the right panel.

Therefore, our subsequent study, which was specifically aimed at extracting and comparing the characteristic learning times τ, involved a longer learning time combined with a higher sampling rate (Huys et al., 2004). Again, the strength of frequency locking ($\psi^{\mathrm{pc}}_{\mathrm{balls}}$) and phase variance between ball movements ($\sigma\Theta$) saturated monotonically, whereas the number of consecutively thrown balls increased explosively. All these evolutions could be cast in exponential form. [2]

Interestingly, the time scales on which these processes evolved were ordered identically across participants: the degree of frequency locking ($\psi^{\mathrm{pc}}_{\mathrm{balls}}$; reflecting the global temporal constraint) always increased faster than the degree of phase locking ($\sigma\Theta$; reflecting the local temporal constraint), which, in turn, always evolved faster than the outcome variable, the number of consecutive throws. (See also Fig. 2; in the first panel this distinct evolution is clearly visible; in the second and third panel it is less evident, but nevertheless present). This finding suggested that learning to juggle involves a temporal hierarchy. Another intriguing finding was that in about half of the participants (6 out of 13), the strength of frequency locking ($\psi^{\mathrm{pc}}_{\mathrm{balls}}$) deteriorated prior

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[2] In some participants, however, the degree of frequency and phase locking evolved too erratically to reliably extract time scales. The data of these participants were excluded from this analysis; see Huys et al. (2004).

to its exponential saturation (in some participants, the phase variance, $\sigma\Theta$, decreased as well; see Fig. 2). In an attempt to uncover the origin of this initial deterioration in performance, we first investigated the structure of the entire 9D-juggling pattern (3 balls × 3 directions) using PCA. As expected, the dimensionality of the pattern reduced in the course of learning: the variance accounted for by the first two modes increased on average from 70% to 78% as learning progressed. The examination of the eigenvector spectra of the ball trajectories as a function of time showed that the contribution of the vertical components of the ball trajectories to the first two modes increased, whereas the contribution of the other two directions decreased. In other words, the juggling pattern became more and more confined to a single plane (see Fig. 3). In a few participants this spatial reorganization of the juggling pattern was so pronounced that an abrupt switch was visible in the spectral content of the projections of the first modes: initially the main frequency of the horizontal ball movements covered most of the variance, whereas after the switch the variance of the entire pattern was dominated by the main frequency of the vertical ball movements. However, this spatial reorganization could not account for the initial deterioration in performance, as this latter phenomenon was only present in one of the six participants who showed an abrupt spatial reorganization. To further examine the initial deterioration in performance, we computed the covariance function between the evolution of the frequency locking strength ($\psi_{x,y}^{\mathrm{pc}}$) and the relative phase variance ($\sigma\Theta$).

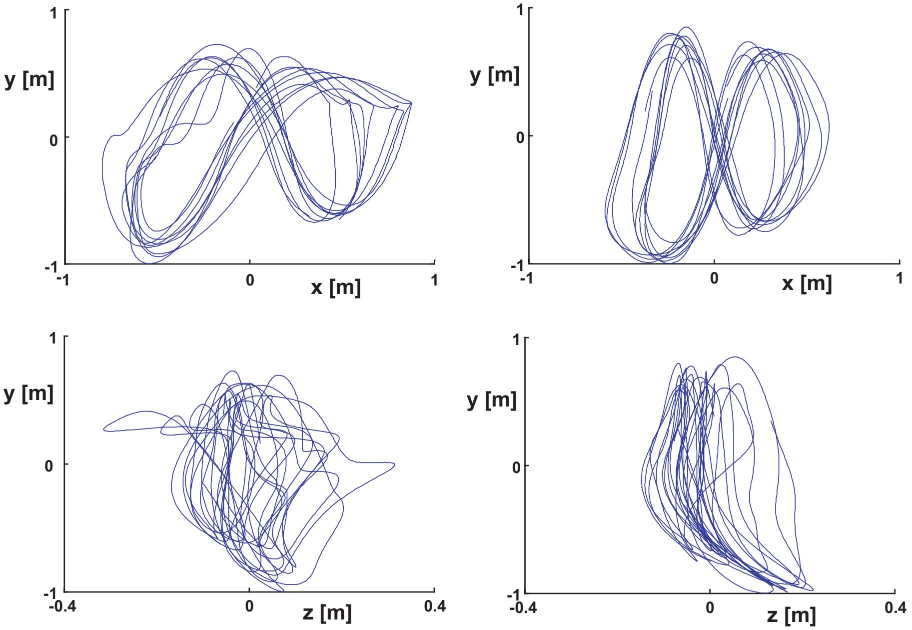

Fig. 3. Examples of the ball trajectories in the *xy*-direction (upper panels) and *yz*-direction (lower panels) early (left column) and late in learning (right column) for a single participant. Notice that in both planes the juggling is more regular and that the relative contribution of the *x*- and *z*-direction decreased in the course of learning.

Its (absolute) maximal value occurred either at lag-zero or lag-one; in other words, the initial deterioration in the frequency locking strength ($\psi^{\rm pc}_{x,y}$) occurred simultaneously with, or was preceded by, an increase in the relative phase variance ($\sigma\Theta$). This result strengthened the suggestion that a temporal hierarchy was present in learning: apparently, a transient deterioration in performance was the price to be paid for a violation of the stipulated order.

The exponential saturation of the frequency locking strength and relative phase variance discussed in the preceding suggests that pronounced improvements in juggling performance, in terms of pattern stability, only occur relatively early in learning (Huys et al., 2003, 2004). This suggestion might be deceptive, however. When comparing the performance of intermediately skilled jugglers with that of expert jugglers with years of practice, we observed that the experts had higher frequency locking strengths ($\psi^{\rm pc}_{\rm balls}$) than the intermediates (Huys & Beek, 2002). Furthermore, unlike the intermediately skilled jugglers, the performance of the experts did not worsen when juggling at non-preferred tempos, even though they performed over a larger range of juggling tempos than the intermediately skilled jugglers. Recall that Post et al. (2000) showed that juggling at a faster than preferred speed is accompanied with an increase in dimensionality of the control structure, which makes the solid performance of experts at different tempos all the more remarkable.

In sum, although only some events pertaining to the ball movements became more regular in the course of learning, the overall stability of performance improved monotonically and the 9D ball pattern became more confined to a single plane. Improvement of the goal behavior and the stability of performance occurred at different, invariantly ordered (i.e., ordinally fixed), rates of change, suggesting that the acquisition of juggling progresses according to a temporal hierarchy. The stability of performance increases continually during extended times of practice (years) and allows for robust, solid control across a wide range of tempos.

3.3. Postural sway in juggling

Whereas postural sway evolves erratically during quiet stance (Collins & De Luca, 1995; Frank, Daffertshofer, & Beek, 2001; Newell, Slobounov, Slobounova, & Molenaar, 1997), several experimental studies have shown that supra-postural tasks may impose a task-specific structure onto postural sway patterns (Bardy, Marin, Stoffregen, & Bootsma, 1999; Bardy, Oullier, Bootsma, & Stoffregen, 2002; Marin, Bardy, & Bootsma, 1999). Furthermore, comparisons of expert and non-expert rifle shooters revealed that the learning of this supra-postural task led to a reduction of sway variability (Era, Konttinen, Mehto, Saarela, & Lyytinen, 1996). In this study, only the non-experts' shooting performance correlated negatively with sway variability, but since the sway patterns were not analyzed further, it is not clear whether the experts had also adopted a specific coordination between shooting and swaying, or whether they had merely "fixated" the center of mass above the base of support.

Our investigations of both postural sway components during juggling showed that sway amplitude as well as its coordination with the juggling pattern change during learning. In both studies (Huys et al., 2003, 2004), we found instances in which postural sway was coordinated with the ball movements in terms of 3:1 and 3:2 frequency locking ratios as well as switches between them, reflecting period doubling (see Fig. 4). These patterns were often embedded in low frequency evolutions, which sometimes dominated the sway trajectories. Considering that a single ball cycle implies equivalent circulations of the other two balls, and that each ball cycle in the vertical direction requires one arm oscillation, it is apparent that a 3:1 (3:2) locking ratio with the ball trajectories in the vertical direction indicates that as one arm (both arms) oscillates once, a single sway cycle is completed. To further understand these locking ratios as well as the transitions between them, we developed a two-segment, unidirectional 3D mechanical model for the expression of the arm movements in the sway cycles (see the Appendix in Huys et al., 2003). In brief, the model describes the (force) projections onto the horizontal support plane as a function of the angles of forearm and upper arm rotations, and shows how small deviations in the amplitudes and symmetries herein may change the relative contribution onto the horizontal of the first and second harmonic in the projections. According to this model and consistent with the data, the 3:2 locking ratio was more likely to occur than the 3:1 locking ratio, especially for the anterior–posterior sway direction (AP-sway). Interestingly, the model showed how relatively small temporal deviations from perfect symmetry between the arm oscillations, and/or the magnitude and offset of the angles involved may induce a switch between both coordination modes *without* affecting the degree to which the prevailing task constraints of juggling are satisfied. In other words, motor equivalence may result from the manner in which postural sway becomes embedded into the juggling machinery.

Recall that, in the course of practice we found for both sway directions a variety of dynamical fingerprints: stable solutions (i.e., 3:1 and 3:2 frequency locks) emerged and sometimes disappeared, while occasionally switches between solutions occurred (Huys et al., 2003, 2004). Given that the stability of juggling performance improved

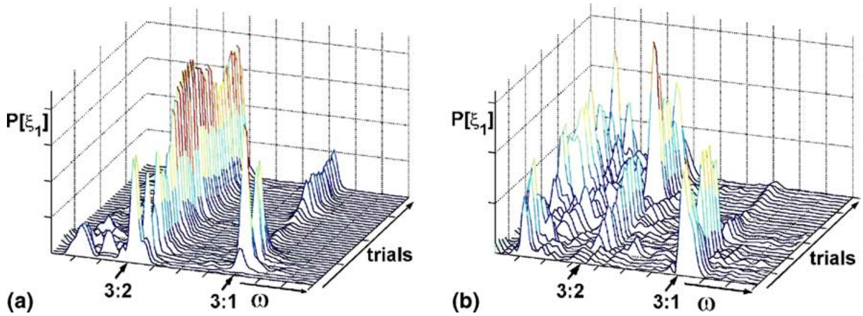

Fig. 4. Power spectral densities ($P[\xi]$) of the projections onto the first mode of AP-sway for the windowed PCA (window size 5) as a function of practice for two participants. The short arrows on the frequency axis (ω; horizontal axis) in each panel indicate the frequency corresponding to the 3:1 and 3:2 frequency locking ratio of the first mode of AP-sway with the ball trajectories. The long arrow indicates practice time (in trials). Notice the switch from the 3:1 to the 3:2 locking ratio early in learning.

monotonically, it follows that, as postural sway became embedded in a task-specific fashion, it maintained a degree of autonomy relative to the overall functional organization. In order to gain further insight into the task-specific embedding of posture during practice, Huys et al. (2004) quantified the characteristic times τ of the changes in the coordination with the ball movements, distinguishing between "amplitude effects" ($\psi_{x,y}^{\mathrm{amp}}$; i.e., by computing the 3:2 frequency locking strength without rescaling the time-series, thus concentrating predominantly on the amplitude of the locked oscillations) and "pure coordination effects" ($\psi_{x,y}^{\mathrm{amp}}$; i.e., by rescaling the time-series to unit variance, thus focusing on coordinative tendencies alone). In the course of learning, the amplitude of the 3:2 frequency locked oscillations ($\psi_{\mathrm{sway}}^{\mathrm{amp}}$) for both sway directions decreased at a time scale similar to that of the decreasing variance of the relative phase between the ball trajectories ($\sigma\Theta$), although for the medio-lateral direction (ML-sway) also non-exponential decreases were observed. These results were confirmed by the correlation analysis between the learning curves of sway ($\psi_{\mathrm{sway}}^{\mathrm{amp}}$) and the variance of the relative phase between ball trajectories ($\sigma\Theta$): for both directions positive correlations of around 0.5 were found, indicating that large sway amplitudes went hand in hand with a large relative phase variance (i.e., "poor" performance) – recall that this variance decreased in the course of learning. In fact, further hints that the amplitude of the AP-sway oscillations may affect juggling performance were found in the small sample of participants whose performance initially deteriorated. The covariance function between the evolution of the frequency locking strength between ball trajectories ($\psi_{\mathrm{balls}}^{\mathrm{pc}}$) and the AP-sway amplitude ($\psi_{\mathrm{AP\text{-}sway}}^{\mathrm{amp}}$) was maximal (in absolute terms) at either lag-zero or lag-one, which indicated that (non-monotonic) changes of this sway component either coincided or preceded this initial deterioration in the stability of juggling performance.

The pure coordination, however, evolved in a markedly different fashion. The strength of the 3:2 frequency locking of ML-sway ($\psi_{\mathrm{ML\text{-}sway}}^{\mathrm{pc}}$) often changed as learning progressed, but these evolutions were often not exponential and no consistent pattern across participants was found. In contrast, the strength of the 3:2 locking mode for AP-sway ($\psi_{\mathrm{AP\text{-}sway}}^{\mathrm{pc}}$) was rather steady throughout learning, or, when expressed in terms of characteristic times τ, evolved orders of magnitudes slower than the goal behavior (up to years, as indicated by $\tau_{\mathrm{AP\text{-}sway}}/\tau_{\sigma\Theta}$ values of around 10^{-5} and larger). Furthermore, the evolutions of neither sway direction ($\psi_{\mathrm{sway}}^{\mathrm{pc}}$) correlated with the development of the relative phase variance ($\sigma\Theta$), which suggested that these pure coordination modes are not crucial for juggling performance, or, alternatively, that the requisite changes had already occurred before quantification was possible (see Huys et al., 2004). The mechanical model may (partly) account for the observed differences between the ML- and AP-sway evolutions. It showed that ML-sway is more vulnerable to small deviations in relevant parameters (e.g., the degree of symmetry between the arm movements) than AP-sway. In addition, the arm movement-induced projections onto the horizontal plane readily transferred to AP-sway via rotations in the ankles, hips and knees, which was not the case for ML-sway.

In sum, notwithstanding the large variability in the sway patterns, clear evidence was found that sway became coordinated to the supra-postural juggling task in terms of 3:2 and 3:1 frequency locking ratios with the ball movements. These coordination modes may reflect the mechanical consequences of the oscillating arms, although the derived model cannot account for the observed amplitude reduction. In the course of learning, initially stable frequency locking solutions emerged and sometimes switches between solutions occurred. As learning progressed, the amplitude of the sway patterns decreased, often at a time scale similar to that of the relative phase variance. In combination, the results indicated that postural sway maintains a degree of autonomy relative to the overall functional organization in which it becomes embedded, and that, at least on the time scale of the experiments, functional stability is achieved more by minimizing the variance with regard to the base of support than by changing the dynamics of the patterns involved.

3.4. Visual perception in juggling

Obviously, successful juggling requires the juggler to continually pick-up information regarding the whereabouts of the balls. Since the balls are mostly airborne, optical information about their flight trajectories is crucial. However, the juggler is confronted with the need to divide his or her visual attention over multiple balls. In order to examine which part of the ball trajectory jugglers prefer to look at, Van Santvoord and Beek (1994) introduced an occlusion method that allowed the (intermediately skilled) jugglers to self-select the moments of (non-)occlusion. Specifically, the participants in this study juggled the 3-ball cascade while wearing liquid crystal glasses that rhythmically opened and closed at preset intervals. By adjusting the frequency and phasing of juggling to the intermittent viewing windows, the participants could thus select the part of the ball trajectories they preferred to watch. In most trials, no phase locking between the opening and closing of the glasses and juggling was achieved, implying that the pick-up of optical information occurred at various parts of the ball trajectories. However, in the trials in which phase locking was achieved, the phasing of the juggling was adjusted to the rhythm defined on the glasses so that the part of the ball trajectory just after the zenith was observed. In these trials the variability between consecutive throws and catches was smaller than in the other trails. Together, these results suggested that, although the zenith may be of special importance, many parts of the balls trajectories are sufficiently informative to sustain juggling.

In fact, as novices start out to learn to juggle, task-specific coordination between eye and ball movements is usually absent (in terms of $\psi^{\mathrm{amp}}_{\mathrm{eyes}}$; Huys et al., 2004). Given the considerable, albeit restricted, amplitude of the eye movements, this implies that the point-of-gaze wanders "all over the place". After elimination of these amplitudes effects by normalizing the time-series to unit variance, a 3:2 frequency locking between the vertical component of the eye movements ($\psi^{\mathrm{pc}}_{\mathrm{eyes},y}$) and the ball trajectories often emerged early in learning (if it was not already present), with a strength that remained steady after the locking was established (see Fig. 5). Surprisingly, the 3:1 frequency locking was never dominant in the vertical direction. In contrast, for the horizontal component of the eye movements the latter 3:1 coordination ($\psi^{\mathrm{pc}}_{\mathrm{eyes},x}$) increased exponentially, albeit with variable time scales across participants.

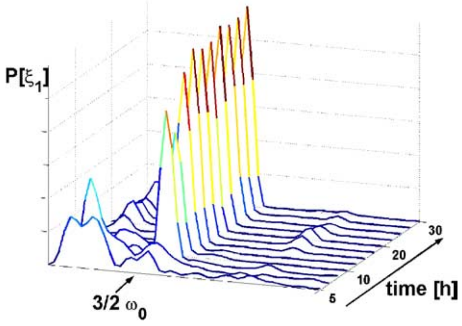

Fig. 5. Power spectral densities ($P[\xi]$) of the projections onto the first mode of the eye movements in the vertical direction for the windowed PCA (window size 3) as a function of practice in one participant. The short arrow on the frequency axis (ω; horizontal axis) indicates the frequency corresponding to the 3:2 frequency locking ratio of the first mode of the eye movements in the vertical direction with the ball trajectories. The long arrow indicates practice time (in hours). Notice the emergence of the 3:2 locking ratio early in learning.

Note that this implies that the learners increasingly coordinated their (horizontal) eye movements to the consecutively tossed balls. Furthermore, the evolution of this 3:1 locking correlated on average negatively with the phase variance ($\sigma\Theta$; $r \approx -0.36$; the correlations with the vertical component as well as the amplitude effects of both components vanished), confirming its increase in the course of learning and suggesting that it is important for neophyte jugglers to establish. In fact, this correlation was more pronounced in the fast learners ($r \approx -0.52$) but absent in the slow learners. It may be speculated that the importance of this coordination mode resides in synchronizing extra-retinal information to optical information, which may be more easily established once the head is stabilized in space, which, in terms of amplitude effects ($\psi^{\mathrm{amp}}_{\mathrm{head}}$), often occurred at very fast rates (Huys et al., 2004). In sum, these results suggest that especially the embedding of the horizontal components of the eye movements in the functional organization of juggling, serving the pick-up of relevant optical information, is an essential aspect of learning to juggle.

In addition to this longitudinal learning study, we examined point-of-gaze movements as a function of expertise, tempo and juggling pattern (Huys & Beek, 2002). Intermediately skilled and expert jugglers juggled both the 3-ball standard cascade and the reverse cascade (see Fig. 5) at three distinct self-selected tempos (fast, preferred and slow). The experts showed a stronger degree of frequency locking between the ball trajectories ($\psi^{\mathrm{pc}}_{\mathrm{balls}}$) than the intermediately skilled jugglers. Point-of-gaze was often coordinated to the ball movements in a 3:1 frequency locking ratio, indicating that a gaze shift was made to each consecutively thrown ball. The vertical component of the point-of-gaze, however, was sometimes coupled to the ball movements with a 3:2 ratio, indicating that in this direction a gaze shift was made to every second ball. Although the strength of these mode locks (in term of $\psi^{\mathrm{pc}}_{\mathrm{eyes}}$) did not differ significantly between skill groups, the incidence of frequency locking did. The experts more often adopted a so-called "gaze through", a behavioral mode in which the point-of-gaze remains bounded within a very small area within the juggling pattern (see Fig. 6). Although frequency coordination between the point-of-gaze and the ball movements was usually absent during a gaze through, this was not always the case in the experts. Furthermore, rescaled to the size of the ball patterns, and across conditions, the gaze amplitudes of the experts were almost half the size of those of the intermediately skilled jugglers, although only in the fast condition the amplitude effect was significant. The experts also tended to intersect their line of gaze less often with the balls, except when juggling fast. In combination, these results were taken to imply that the experts relied less on foveal information, which is consistent with anecdotal evidence suggesting that only experts are able to easily switch back and forth between visual tracking of the balls and the gaze through and to juggle even blindfolded. These findings were further interpreted to imply that in the course of learning the dependency on foveal information decreases, probably due to an increased reliance on kinesthetic, haptic and peripheral visual information by using a so-called "visual pivot", that is, by diffusing and/or shifting attention across the visual periphery without changing gaze (Williams & Davids, 1998). All in all, it is evident that the organization of a juggler's gaze is affected by a myriad of constraints, some without a direct bearing on vision.

Thus, during practice a specific coordination between eye and ball movements is established within which the 3:1 locking of the horizontal eye movements seems to be of special importance. Extended practice may promote reliance on multiple sources of information, allowing the expert juggler to switch flexibly between functional organizations involving distinct perceptual systems.

3.5. Breathing while juggling

Synchronization between breathing and locomotion, generally referred to as locomotor–respiratory coupling (LRC), has been observed in a variety of activities,

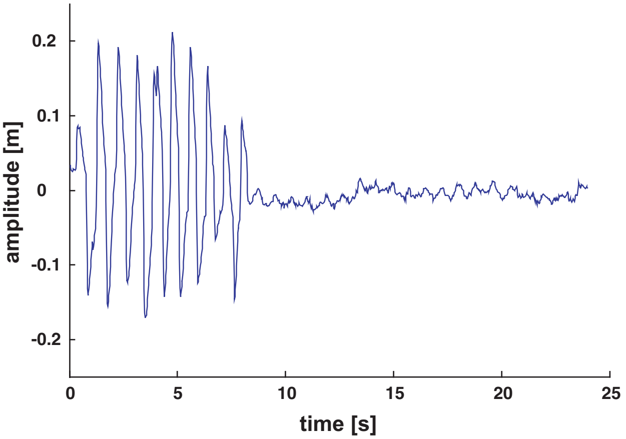

Fig. 6. Example of point-of-gaze settling into a gaze through.

including running (Bernasconi & Kohl, 1993; Bramble & Carrier, 1983), cycling (Bonsignore, Morici, Abate, Romano, & Bonsignore, 1998) and rowing (Mahler, Shuhart, Brew, & Stukel, 1991; Siegmund et al., 1999). Explanations of LRC have been sought predominantly in terms of the locomotion-induced mechanical impacts onto the thoracic complex (Bramble & Carrier, 1983; Bramble & Jenkins, 1993) and in terms of efficiency (Bernasconi & Kohl, 1993; Garlando, Kohl, Koller, & Pietsch, 1985). Recently, however, Amazeen, Amazeen, and Beek (2001) have documented LRC during wheelchair propulsion, a type of locomotion with only moderate metabolic demands and mechanical impacts. They therefore proposed instead that LRC might be based on coordinative principles. This suggestion prompted us to study breathing in juggling because the metabolic demands and mechanical impacts of juggling are arguably even smaller than in manual wheelchair propulsion (Huys et al., 2003). Although sometimes incidences of specific p:q locking ratios were observed (i.e., 1:1, 3:4 2:3, 2:5 and more), no systematic changes in coordination were found. We therefore concluded that the synchronization of breathing and arm movements is of minor importance in juggling. [3] In retrospect, this result may be understood in view of our recently proposed model accounting for LRC phenomena (Daffertshofer, Huys, & Beek, 2004), which was based on a re-analysis of Siegmund et al.'s experimental data on rowing. In brief, we proposed that the cyclical abdominal pressure (induced mechanically by the rowing movements) modulates the self-sustaining respiration, which causes (local) maxima of the effective value of the oxygen volume in the lungs at frequency ratios between small integers representing both oscillatory processes. Optimizing the oxygen volume may be seen as the driving force underlying LRC. The location and magnitude of the maxima of the effective value depends on the task. For juggling, one may readily expect multiple "neighboring" (local but low) maxima of the effective value of the oxygen volume, rendering the occurrence of consistent frequency and/or phase locking between arm movements and breathing unlikely.

4. Conclusions

Based on previous studies, Beek and Van Santvoord (1992) proposed a 3-stage "model" for the learning of juggling: accommodating Shannon's equation (stage 1), probing for the primary mode lock (in terms of the tiling of the hand-loop time via the dwell time, stage 2), and subsequent discovery of other mode locks corresponding to smaller dwell ratios (stage 3). The crux of this model is that it suggests the existence of a certain temporal order in accommodating the temporal constraints on juggling. In the studies discussed here the partitioning of the hand-loop time was

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[3] Notice that, particularly early in learning, novice jugglers are unable to sustain juggling for long periods of time. Due to the rather low frequency of respiration, the accuracy of the spectral estimates was therefore rather poor. That is, under such circumstances the results pertaining to respiration are ambiguous and should thus be interpreted with caution.

not examined further: Their main focus was on time-continuous measures for characterizing couplings and their evolutions. Nevertheless, the proposed temporal hierarchy in satisfying the global (frequency locking) and local (phase locking) temporal constraints is similar in spirit to the interpretation offered by Beek and Van Santvoord (1992). On the level of the goal behavior, the progression of the number of consecutive throws and the adaptation to two different timing constraints evolved according to a temporal hierarchical order. When this temporal hierarchy was violated, performance deteriorated temporarily. Similarly, different learning effects were found for the spatial/temporal as well as the ball/hand related variables that may allow for a similar treatment, although no such attempt has been made to date (see also Van Santvoord & Beek (1996), who found that the spatial variables, but not the temporal variables, were less variable in the experts than in the intermediately skilled jugglers). In combination, these results suggest that not all variables, as well as the constraints pertaining to them, are equally important to control – a suggestion that is at the heart of the uncontrolled manifold and related notions (see Section 1). That is, the high-dimensional task space of 3-ball cascade juggling seems to be stratified according to functional relevance. The gradual, rate-dependent accommodation of at least a subset of the constraints on juggling suggests that, ideally, learning progresses hierarchically in accordance with this stratification.

Tentatively, the evolution of the subsystems subserving juggling performance could be interpreted along similar lines. For instance, the amplitude of the frequency locked oscillations of postural sway reduced at times scales similar to that of the phase variance, unlike the pure coordination. Large sway amplitudes appear to jeopardize continuation of supra-postural tasks like juggling more than negligible sway amplitudes; thus, the structure in the latter may be less important. In contrast, it seems essential to establish a pure coordination of the eye movements to the ball trajectories, possibly to synchronize extra-retinal information to optical information. Months if not years of training seem to be required to learn to rely on peripheral vision and kinesthetic and haptic information and, to a degree, to "liberate" oneself from the need to direct foveal vision toward the ball trajectories, as suggested by the development of a "gaze through" or "visual pivot". Ultimately, successful reliance on several sources of information may allow for dynamically distinct organizations among which the juggler may flexibly switch. Clearly, this flexibility is reminiscent of Bernstein's (1996) proposal of a delegation of control, although, as it stands, "distribution of control" may be more adequate, as this notion does not necessitate embracing Bernstein's (1996) notion of levels.

In sum, the learning of a complex perceptual–motor skill like juggling is not a singular, compulsory process in which all the contributing components evolve along fixed lines. On the contrary, although a single behavioral goal may be achieved in an increasingly successful manner, the functional and dynamical relations among the subsystems underlying the goal behavior manifest a rich variety of dynamical fingerprints in terms of characteristic times and forms of change, as anticipated by Newell et al. (2001). The assembly and embedding of subsystem, as well as their contribution to the goal behavior, depend on specific organismic and task constraints, as was already intuited by Bingham (1988) in his distinction between inherent and incidental dynamics. Practice-induced changes in the goal behavior involve, at least to some extent, a temporal hierarchy, which parallels, by hypothesis, the stratification of constraints at hand.

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